Hawk07-C-Dyn-v2: Quantum Recovery on an Explicitly Dynamical Conical-Formation Spacetime

Exact Einstein Source, Causal Hosting, Finite Certificates, and Classical-Return Limits

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Abstract

We replace the static prescribed conical host of Hawk07-C by an explicit smooth, time-dependent spacetime. The metric combines a settling spatially flat FLRW scale factor with a retarded cylindrical deformation. The deformation vanishes outside its radial light front, produces a smooth finite-radius string-like core, and becomes exactly conical on every bounded exterior region after a finite settling time. We compute its complete Einstein tensor and thereby give an exact Einstein–effective-source pair with conserved stress-energy. The full regular-core spacetime is globally hyperbolic and has no everywhere timelike Killing field. The source is engineered rather than derived from a specified matter action; its outgoing transition violates the null energy condition. Thus this exact construction is not presented as an ordinary positive-energy cosmic-string formation solution.

The finite quantum recovery construction is retained and proved independently of the geometry. Its complementary Haar moment gives the sufficient released-dimension law

$$r_* \geq \frac{4d^2\sqrt{f}}{\varepsilon^2\sqrt{p}}$$

for full-diamond return error at most ε, outside a device set of measure at most p. One specified decoder core works at all later retained cuts. For $d = 16$, a family of measure at least 1/64 combines accurate return, strict three-cut coherent entropy turnover, and a one-qubit final residual.

Under explicit ideal-control assumptions, these channels have finite causal realizations on the evolving host, including realizations in which the payload is present during the nonstationary phase. Geometry changes causal schedules and proper-time calibration; it does not by itself define a quantum noise channel. Physical propagation errors require a separately specified implementation map and error allowance. No geometric wormhole, material transport mechanism, or quantum gravitational backreaction is constructed.

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Status of the conversion. This is a replacement of the host geometry and its hosting theorem, not a relabelling of a static metric. The new host has nonzero, time-dependent curvature during formation, an explicitly calculated Einstein source, and a late conical exterior.

Physical qualification. The exact source below is obtained by an inverse Einstein construction: first specify a smooth metric, then calculate $T_{\mu\nu} = G_{\mu\nu}/\kappa$. This supplies an exact classical metric–source pair, but not an initial-value solution of an independently specified microscopic matter theory. In particular, conservation alone does not establish a causal matter evolution law or physical realizability.

Scope, dependencies, and the cosmic-string comparison

The supplied Hawk07-C manuscript [1] separates finite reference-preserving quantum recovery from its prescribed static conical host. We preserve that separation while replacing the host by a genuinely evolving spacetime.

There are three logically distinct components:

1. an explicit smooth Lorentzian metric and its exact Einstein source;

2. finite-dimensional encoding, access, decoding, and leakage statements;

3. a conditional implementation theorem connecting those channels to causally scheduled ideal controls. Neither a recovery channel nor a destructive measurement determines the metric. Conversely, a metric does not automatically supply finite quantum registers, identity wires, or a decoder.

Relation to conventional cosmic-string formation

Lake and Suyama study the perturbative formation of a straight string in an FLRW universe, including compensating fluid perturbations, distinct sound and light cones, and a gravitational shock [2]. In their weak-field conventions $c = 1$, a string with tension qU has deficit $4\pi GU(1 + q)$; for $q \neq 1$ its near-string metric also has a nontrivial gravitational potential. An expanding FLRW factor does not disappear merely by taking a near-string limit.

Our exact construction is different: it uses a smooth finite core and a smooth outgoing transition, not a thin source with a distributional shock. No sound horizon is assigned, because no perfect-fluid equation of state is imposed. A settling scale factor is chosen explicitly so that the late exterior really is static. The cost is an engineered transition source with negative-energy regions. We do not identify this model with the conventional formation solution.

External mathematical tools

The finite proofs use the information–disturbance theorem [4], diamond-norm semidefinite programming [6], and

finite unitary synthesis [7]. The remaining displayed geometric and finite-moment calculations are given locally.

No publication status or theorem from an unavailable companion manuscript is required. The supplied source remains the documentary record for its own companion comparisons.

An explicit evolving spacetime

Smooth switching functions and the core profile

Define

$$b(x) = \begin{cases} e^{-1/x}, & x > 0, \\ 0, & x \le 0, \end{cases} \qquad S(x) = \frac{b(x)}{b(x) + b(1-x)}.$$

Then S is smooth,

$$S(x) = 0 \quad (x \le 0), \qquad S(x) = 1 \quad (x \ge 1), \qquad S'(x) > 0 \quad (0 < x < 1),$$

and

$$S(1-x) = 1 - S(x), \qquad S''(1/2) = 0.$$

Fix

$$0 < r_0 < r_1, \quad 0 < \beta < 1, \quad \lambda > 0, \quad \tau_a > 0, \quad \tau_f > 0, \quad c > 0.$$

Set

$$q_{\mathrm{c}}(r) = S\left(\frac{r - r_0}{r_1 - r_0}\right),$$

$$Q_{\mathrm{c}}(r) = \int_0^r q_{\mathrm{c}}(s)\,\mathrm{d}s,$$

$$\rho_{\mathrm{c}} = r_1 - Q_{\mathrm{c}}(r_1),$$

$$\alpha_\infty = 1 - \beta.$$

Consequently,

$$0 \le Q_c(r) \le r, \qquad Q_c(r) = 0 \quad (r \le r_0),$$

and

$$Q_c(r) = r - \rho_c \quad (r \ge r_1).$$

Use conformal time η, and define

$$a(\eta) = 1 + \lambda S(\eta/\tau_a), \qquad a_\infty = 1 + \lambda,$$

$$u = \eta - r/c, \qquad s(u) = S(u/\tau_f),$$

$$F(\eta, r) = r - \beta Q_c(r) s(u).$$

The metric and the included axis

Let

$$\widetilde{M} = \mathbb{R}_\eta \times \mathbb{R}^2_{x,y} \times \mathbb{R}_z.$$

For $r > 0$, use the ordinary polar coordinates

$$x = r\cos\varphi, \qquad y = r\sin\varphi, \qquad \varphi \in \mathbb{R}/2\pi\mathbb{Z}.$$

The metric is

$$\text{boxed}\, g = a(\eta)^2[-c^2 d\eta^2 + dr^2 + F(\eta, r)^2 d\varphi^2 + dz^2].$$

The axis is included. Because $F = r$ for $r \le r_0$, the polar coordinate degeneracy there is the ordinary removable coordinate degeneracy of a smooth FLRW metric. No conical singularity or distributional apex is inserted into $\widetilde{M}$.

Proposition (Smoothness, signature, and uniform bounds). *The metric extends smoothly across the axis, has signature* $(-+++)$*, and satisfies*

$$\alpha_\infty r \le F(\eta, r) \le r.$$

Writing

$$h_\eta = \mathrm{d}r^2 + F(\eta, r)^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad h_{\mathrm{E}} = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2,$$

one has

$$\alpha_\infty^2 h_{\mathrm{E}} \le h_\eta \le h_{\mathrm{E}}.$$

Proof. The bounds follow from $0 \le Q_{\mathrm{c}} \le r$ and $0 \le s \le 1$. Thus $F > 0$ for $r > 0$. Near the axis $F = r$ exactly, so the metric is smooth in Cartesian coordinates. The remaining assertions follow from the diagonal form and the angular coefficient bounds.

Causal formation rather than simultaneous global switching

Proposition (Retarded support and null fronts). *For $u \le 0$, the metric is exactly the spatially flat FLRW metric with scale factor $a(\eta)$. The conical deformation therefore vanishes outside $r < c\eta$.*

The hypersurfaces $u =$ constant, *away from the axis, are null. The transition occupies*

$$0 < \eta - r/c < \tau_f.$$

For $\eta \le 0$, the entire spacetime slice is Minkowskian.

Proof. If $u \le 0$, then $s = 0$ and $F = r$. Moreover,

$$g^{-1}(\mathrm{d}u, \mathrm{d}u) = a^{-2}\left(-c^{-2} + c^{-2}\right) = 0.$$

For $\eta \le 0$, both switches vanish.

The string-induced geometric deformation has causal support relative to the initially straight axis. This support statement is not, by itself, a derivation of a hyperbolic matter evolution equation. The FLRW factor is independently specified background evolution.

Exact late conical exterior

Proposition (Finite settling on bounded exterior regions). *Consider a region with*

$$r_1 < r \le R < \infty.$$

If

$$\eta \geq \eta_{\mathrm{set}}(R) := \max\{\tau_a, \tau_f + R/c\},$$

then $a = a_\infty$ *and*

$$F = \alpha_\infty r + \beta \rho_{\mathrm{c}}.$$

The coordinate changes

$$T = a_\infty \eta, \qquad \bar{r} = a_\infty \left(r + \frac{\beta \rho_{\mathrm{c}}}{\alpha_\infty} \right), \qquad \bar{z} = a_\infty z$$

put the metric on that region in the form

$$g = -c^2 \mathrm{d}T^2 + \mathrm{d}\bar{r}^2 + \alpha_\infty^2 \bar{r}^2 \mathrm{d}\varphi^2 + \mathrm{d}\bar{z}^2.$$

The exterior deficit is

$$\Delta_\infty = 2\pi(1 - \alpha_\infty) = 2\pi\beta.$$

Proof. Under , both switches have attained their final values throughout the region. Substitution gives the result.

This is a local exterior identification. The formal cone apex $\bar{r} = 0$ is not part of this exterior chart; the actual axis is replaced by the smooth core.

There is no finite time at which the entire infinite spatial exterior has settled. At every finite η, sufficiently large r still lies outside the front. Settling holds on each bounded exterior region, not uniformly over all radii.

Exact Einstein source and genuine dynamics

We use zero cosmological constant and

$$\kappa = \frac{8\pi G}{c^4}, \qquad G_{\mu\nu} = \kappa T_{\mu\nu}.$$

The curvature convention is the one for which spatially flat FLRW has positive $G_{\hat{0}\hat{0}}$ when its expansion rate is nonzero.

A compact exact formula

Put $x^0 = c\eta$, and write

$$F_0 = \frac{\partial F}{\partial x^0}, \qquad F_{00} = \frac{\partial^2 F}{\partial (x^0)^2}, \qquad F_{0r} = \frac{\partial^2 F}{\partial x^0 \partial r}.$$

Define

$$\mathscr{H} = \partial_0 \log a, \qquad \mathscr{J} = \frac{F_0}{F}, \qquad \mathscr{H}_0 = \partial_0 \mathscr{H}.$$

An orthonormal frame for the physical metric is

$$e_{\hat{0}} = a^{-1}\partial_0, \quad e_{\hat{r}} = a^{-1}\partial_r, \quad e_{\hat{\varphi}} = (aF)^{-1}\partial_\varphi, \quad e_{\hat{z}} = a^{-1}\partial_z.$$

Theorem (Complete Einstein tensor). *The only possibly nonzero orthonormal components of the Einstein tensor are*

$$G_{\hat{0}\hat{0}} = a^{-2}\left[-\frac{F_{rr}}{F} + 3\mathscr{H}^2 + 2\mathscr{H}\mathscr{J}\right],$$

$$G_{\hat{0}\hat{r}} = -a^{-2}\frac{F_{0r}}{F},$$

$$G_{\hat{r}\hat{r}} = a^{-2}\left[-\frac{F_{00}}{F} - 2\mathscr{H}_0 - \mathscr{H}^2 - 2\mathscr{H}\mathscr{J}\right],$$

$$G_{\hat{\varphi}\hat{\varphi}} = a^{-2}\left[-2\mathscr{H}_0 - \mathscr{H}^2\right],$$

$$G_{\hat{z}\hat{z}} = a^{-2}\left[\frac{F_{rr} - F_{00}}{F} - 2\mathscr{H}_0 - \mathscr{H}^2 - 2\mathscr{H}\mathscr{J}\right].$$

In particular,

$$T_{\mu\nu} := \kappa^{-1} G_{\mu\nu}$$

is a smooth exact Einstein source and obeys

$$\nabla^\mu T_{\mu\nu} = 0.$$

Proof. First remove the conformal factor and set

$$g_0 = -(\mathrm{d}x^0)^2 + \mathrm{d}r^2 + F^2\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

For base indices $A, B \in \{0, r, z\}$, the nonzero warped-product connection terms involving the circle are

$$\Gamma^{\varphi}_{A\varphi} = \frac{F_A}{F}, \qquad \Gamma^{A}_{\varphi\varphi} = -F\partial^A F.$$

Direct contraction gives

$$(R_0)_{AB} = -\frac{F_{AB}}{F}, \qquad (R_0)_{\varphi\varphi} = -F\mathrm{Box}_0 F, \qquad R_0 = -2\frac{\mathrm{Box}_0 F}{F},$$

where $\mathrm{Box}_0 F = -F_{00} + F_{rr}$. Therefore, in the g_0 orthonormal frame,

$$(G_0)_{00} = -F_{rr}/F, \quad (G_0)_{0r} = -F_{0r}/F, \quad (G_0)_{rr} = -F_{00}/F,$$

$$(G_0)_{\varphi\varphi} = 0, \qquad (G_0)_{zz} = (F_{rr} - F_{00})/F.$$

For $g = e^{2\omega} g_0$, $\omega = \log a$, substitution of the conformal connection difference gives

$$G[g]_{AB} = G[g_0]_{AB} - 2\nabla_A\nabla_B\omega + 2\omega_A\omega_B + (g_0)_{AB}(2\mathrm{Box}_0\,\omega + (\nabla\omega)^2).$$

Here

$$\mathrm{Box}_0\,\omega = -\mathscr{H}_0 - \mathscr{J}\mathscr{H}, \qquad (\nabla\omega)^2 = -\mathscr{H}^2,$$

and the angular orthonormal Hessian is $-\mathscr{J}\mathscr{H}$. These identities yield –, with the final factor a^{-2} coming from orthonormal-frame rescaling.

Smoothness follows from Proposition . Conservation is the contracted Bianchi identity.

Remark (What has and has not been solved). The metric and source satisfy Einstein's equation exactly. However, the source has not been shown to arise from a specified scalar field, gauge field, string action, or perfect fluid. Equation is an inverse construction, not an independent matter-model existence theorem.

Explicit derivatives and a curvature witness

Primes on s below mean derivatives with respect to u. The derivatives needed in the Einstein tensor are

$$F_{00} = -\frac{\beta Q_c}{c^2} s'',$$

$$F_{0r} = -\frac{\beta}{c}\left(q_c s' - \frac{Q_c}{c} s'' \right),$$

$$F_{rr} = -\beta q_c' s + \frac{2\beta q_c}{c} s' - \frac{\beta Q_c}{c^2} s''.$$

The scalar curvature is

$$R[g] = a^{-2}\left[-2\frac{F_{rr} - F_{00}}{F} + 6(\mathscr{H}_0 + \mathscr{H}^2 + \mathscr{H}\mathscr{J}) \right].$$

After a has settled, and in the exterior $r > r_1$,

$$\text{boxed}\, R[g] = -\frac{4\beta}{c\, a_\infty^2 F} s'(\eta - r/c) < 0 \quad \text{when} 0 < u < \tau_f.$$

The same exterior has $R = 0$ before the front and after its passage. The deformation is therefore not merely a changing coordinate description of the locally flat static cone.

Late string-like core and its energy per unit length

Once both switches have settled at a given radius,

$$F_{rr} = -\beta q_c', \qquad F_{00} = F_{0r} = 0.$$

The source then has

$$T^{\wedge\wedge}_{\mu\nu} = \text{diag}\,(\varepsilon, 0, 0, -\varepsilon), \qquad \varepsilon = \frac{\beta q_c'}{\kappa a_\infty^2 F} \geq 0.$$

It is supported in the finite core mantle $r_0 < r < r_1$. Its axial tension equals its energy per unit length.

Integrating over a physical transverse cross-section gives

$$\mathscr{U} = 2\pi \int_0^\infty \varepsilon\, a_\infty^2 F \, \mathrm{d}r$$

$$= \frac{2\pi\beta}{\kappa} = \frac{c^4\beta}{4G}.$$

Thus

$$\Delta_\infty = \frac{8\pi G\mathscr{U}}{c^4}.$$

Here $\mathscr{U}$ is energy per unit length, not mass per unit length. This convention fixes the powers of c.

The transition violates the null energy condition

Proposition (An explicit negative-energy region). *The formation source does not satisfy the null energy condition everywhere. It also fails the weak energy condition.*

Proof. Choose $r > r_1$ sufficiently large that

$$\eta = r/c + \tau_f/2 > \tau_a.$$

Then $a = a_\infty$, $q_c = 1$, $q_c' = 0$, $s' > 0$, and $s'' = 0$. Equations and give

$$T_{\hat{0}\hat{0}} = -\frac{2\beta s'}{\kappa c\, a_\infty^2 F} < 0.$$

Also $T_{\hat{\varphi}\hat{\varphi}} = 0$ and $T_{\hat{0}\hat{\varphi}} = 0$. The null vector $k = e_{\hat{0}} + e_{\hat{\varphi}}$ therefore satisfies

$$T_{\mu\nu}k^\mu k^\nu = T_{\hat{0}\hat{0}} < 0.$$

Proposition (Compensating curvature and signed energy). *On a fixed-η, fixed-z transverse disk of radius R,*

$$\int K\,\mathrm{d}A = 2\pi(1 - F_r(\eta, R)),$$

where $K = -F_{rr}/(a^2F)$ is its Gaussian curvature. At every finite η,

$$\int_{\mathbb{R}^2} K\,\mathrm{d}A = 0.$$

After a has settled, the corresponding integral of the comoving energy density per unit physical axial length is also zero:

$$2\pi\int_0^\infty T_{\hat{0}\hat{0}}\, a_\infty^2 F\, \mathrm{d}r = 0.$$

Proof. Integrating $-F_{rr}$ gives the disk formula because $F_r = 1$ near the axis. At each finite time, $F = r$ for sufficiently large r, so $F_r = 1$ there as well. When a is constant, $\kappa T_{\hat{0}\hat{0}} = -a_\infty^{-2} F_{rr}/F$, which gives the final assertion.

The positive late core is compensated by the outgoing transition. This explains why the model must not be advertised as positive-energy string creation from vacuum. The smooth transition is a curvature and matter pulse; no vacuum gravitational-wave interpretation is assumed.

Debugging the spatially homogeneous deficit ansatz

Consider instead

$$g_{\mathrm{hom}} = -c^2\mathrm{d}T^2 + \mathrm{d}r^2 + \alpha(T)^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Away from the axis, its nonzero Einstein components include

$$G_{\hat{0}\hat{r}} = -\frac{\dot{\alpha}}{c\alpha r}, \qquad G_{\hat{r}\hat{r}} = G_{\hat{z}\hat{z}} = -\frac{\ddot{\alpha}}{c^2\alpha}, \qquad G_{\hat{0}\hat{0}} = G_{\hat{\varphi}\hat{\varphi}} = 0.$$

Its required source is therefore not confined near the axis. A common $\alpha(T)$ also changes all radii simultaneously rather than implementing a retarded formation front.

A later matching calculation cannot be assumed to repair those facts without solving its own junction and source problem. Equation supplies a different, explicitly retarded construction.

Global causality, proper time, and nonstationarity

Proposition (Causal tangents and proper time). *Choose ∂_η future directed. A nonzero future-causal tangent $v = A\partial_\eta + w$ satisfies*

$$A > 0, \qquad |w|_{h_\eta} \le cA.$$

For a fixed-coordinate laboratory,

$$d\tau = a(\eta)\, d\eta.$$

For a moving laboratory $x(\eta)$,

$$d\tau = a(\eta)\sqrt{1 - \frac{|x'(\eta)|_{h_\eta}^2}{c^2}}\, d\eta.$$

Proof. These follow directly from $g(v,v) = a^2(-c^2A^2 + |w|_{h_\eta}^2)$ and the definition $g = -c^2 d\tau^2$ along timelike curves.

Theorem (Global hyperbolicity with the regular core included). *Every slice* $\eta = \eta_*$ *is a Cauchy hypersurface of* $(\widetilde{M}, g)$. *In particular, the full regular-core spacetime is globally hyperbolic.*

Proof. The time function η has timelike gradient. A causal curve, parametrized by η, obeys

$$|x'(\eta)|_{h_E} \le c/\alpha_\infty$$

by . If its η-range had a finite endpoint, its Cartesian spatial coordinates would be Cauchy and would converge in $\mathbb{R}^3$. The curve could then be extended by a future- or past-directed timelike segment. An inextendible causal curve must therefore have η-range all of $\mathbb{R}$, crossing each slice exactly once.

One can also verify compact causal diamonds directly. A diamond is confined to a bounded conformal-time interval and a bounded Euclidean spatial ball by the same speed bound. Parametrized causal curves have a uniform Lipschitz bound; convergent subsequences remain causal by smoothness of the metric. The diamond is consequently closed in a compact coordinate region. Together with the temporal function, this proves global hyperbolicity.

Proposition (The evolving host is not globally stationary). *There is no everywhere timelike Killing vector field on* $(\widetilde{M}, g)$.

Proof. The scalar curvature vanishes everywhere on every sufficiently early slice, since that region is Minkowski. It is nonzero at points identified in .

If an everywhere timelike Killing field existed, its maximal orbits, viewed as unparametrized curves, would be

inextendible timelike curves. Indeed, a finite conformal-time endpoint would have a finite spatial limit by the causal speed bound, and the smooth nonzero Killing field would extend the orbit through that point. Every orbit would therefore meet an early Cauchy slice. Scalar curvature is constant along Killing orbits, forcing it to vanish everywhere, a contradiction.

Remark (Removing an axis is a different spacetime). If one instead deletes the regular axis, the resulting punctured spacetime has noncompact causal diamonds: radial causal curves approach the missing axis in finite conformal time. Thus the static paper's excision obstruction is recovered after excision, not after including a smooth core.

A distributional thin-string completion is a third construction. Its causal and analytic properties require an explicitly stated singular-spacetime framework. They do not follow from simply saying that an apex is "retained".

A uniform sufficient causal-routing rule

Let $\mathscr{U} \subset \mathbb{R}^3_{x,y,z}$ be an open Euclidean convex laboratory region whose closure stays away from the core. For $x, y \in \mathscr{U}$,

$$\|y - x\|_2 \le c(\eta_1 - \eta_0), \qquad \eta_1 \ge \eta_0,$$

is a sufficient condition for a future-causal route between (η_0, x) and (η_1, y). Indeed, the straight coordinate segment remains inside $\mathscr{U}$, and $h_\eta \le h_{\mathrm{E}}$ bounds its speed.

During formation, these Cartesian coordinates are not a developing isometry for the spatial geometry. The proof uses the metric inequality, not local flatness.

Destructive clocks on the evolving host

Let $C \cong \mathbb{C}^{D_C}$ have reading basis $\{|\tau_j\rangle\}$, and let A contain all surviving quantum registers. Define

$$K_j = \langle \tau_j|_C \otimes I_A,$$

and

$$\mathrm{Enter}_{C}^{A}(X) = \sum_{j} \mathrm{kb}\, jj_{Y} \otimes K_{j} X K_{j}^{\dagger}.$$

The output Y is classical; the clock factor is absent from the returned quantum interface.

Proposition (Complete clock identity). *The map is CPTP and satisfies*

$$\mathrm{Tr}_{Y}\, \mathrm{Enter}_{C}^{A}(X) = \mathrm{Tr}_{C} X,$$

including arbitrary untouched references.

For distinct coordinate labels η_j, exact event calibration at one event e,

$$\mathbb{P}(j) > 0 \;\Rightarrow \eta_{j} = \eta(e),$$

permits only one occurring label. Proper-time calibration instead uses

$$\tau_{\mathrm{lab}}(\eta) = \tau_{\mathrm{lab}}(\eta_{\mathrm{ref}}) + \int_{\eta_{\mathrm{ref}}}^{\eta} a(v)\, \mathrm{d}v$$

for a fixed-coordinate laboratory.

Proof. The recorded Kraus operators are $|j\rangle_{Y} \otimes K_{j}$, and their adjoint products sum to identity. The sum of the diagonal clock blocks is $\mathrm{Tr}_{C} X$. A reference does not change the calculation. Distinct exact labels give the calibration assertion, and gives the proper-time formula.

Corollary (Independent readout does not change the payload channel). *An independently prepared clock, with no payload-dependent interaction and no subsequent payload control based on its record, can be read destructively and ignored without changing the payload channel.*

Proof. For a product input $\chi_{C} \otimes X$, the surviving payload blocks sum to $\mathrm{Tr}(\chi_{C}) X = X$, also with references.

Clock consumption removes a finite quantum access right. It does not remove η, proper time, the scale factor, or the classical Einstein source.

A clock used to control payload dynamics is not independent in the sense of this corollary. Its coupling, calibration error, records, and environment must be included in the implementation model.

Complete quantum-return interfaces

All quantum spaces below are finite dimensional. Define

$$d_{\mathrm{tr}}(\rho,\sigma)=\frac{1}{2}\|\rho-\sigma\|_1, \qquad d_{\diamond}(\Phi,\Psi)=\frac{1}{2}\|\Phi-\Psi\|_{\diamond}.$$

The diamond norm includes untouched references. For channel differences, reference dimension equal to the input dimension suffices [5].

Definition (Complete encoding and return certificate). A complete encoding is an isometry

$$V: L \to R\otimes B, \qquad \dim L = d \geq 2,$$

with

$$\mathscr{N}=\mathrm{Tr}_B \mathrm{Ad}_V, \qquad \mathscr{C}=\mathrm{Tr}_R \mathrm{Ad}_V.$$

Every inaccessible purifier, lost mode, apparatus environment, and record store required by the encoding realization belongs to B. An external payload reference is not granted to the receiver.

A return certificate specifies a deterministic CPTP decoder $\mathscr{D}: R \to L_{\mathrm{out}}$ and a full-diamond allowance

$$\|\mathscr{D}\mathscr{N}-\mathrm{id}_L\|_{\diamond}\leq\varepsilon.$$

It has a positive physical residual when $\dim B \geq 2$.

For a state τ, put

$$\mathscr{T}_\tau(X)=\mathrm{Tr}\,(X)\tau,$$

and define

$$\varepsilon_{\mathrm{rec}}(\mathscr{N})=\inf_{\mathscr{D}\ \mathrm{CPTP}} d_{\diamond}(\mathscr{D}\mathscr{N},\mathrm{id}_L),$$

$$\delta_{\mathrm{full}}(\mathscr{C})=\inf_{\tau\in\mathscr{S}(B)} d_{\diamond}(\mathscr{C},\mathscr{T}_\tau).$$

Write

$$a_d = 1 - d^{-2}, \qquad b_d = 1 - d^{-1}.$$

The residual here is the complete complement before decoding. Additional discarded decoder systems belong to an enlarged postdecoder complement.

Complete restoration and exact correction

Proposition (Restoration from the entire output). *For any fixed state* ω_L,

$$\mathscr{L}_V(Y) = V^\dagger Y V + \mathrm{Tr}\,((I - VV^\dagger)Y)\omega_L$$

is CPTP and left-inverts Ad_V. *Isometric conjugation preserves trace norms, also after reference extension.*

Proof. Both summands are completely positive, their traces sum to $\mathrm{Tr}\,Y$, and substitution of $VXV^\dagger$ returns X. An isometry preserves all nonzero singular values.

Theorem (Exact full-code correction). *There is a CPTP decoder with* $\mathscr{D}\mathscr{N} = \mathrm{id}_L$ *if and only if*

$$\mathscr{C} = \mathscr{T}_{\tau_B}$$

for some state τ_B. *If* $s = \mathrm{rank}\,\tau_B$, *then* $\dim R \geq ds$, *and there is an isometry* $T : L \otimes \mathbb{C}^s \to R$ *such that*

$$\mathscr{N}(X) = T(X \otimes \sigma)T^\dagger$$

for a state σ *on* $\mathbb{C}^s$. *An exact decoder on all of* R *is*

$$\mathscr{D}(Y) = \mathrm{Tr}_{\mathbb{C}^s}(T^\dagger Y T) + \mathrm{Tr}\,((I - TT^\dagger)Y)\omega_L.$$

Proof. Dilate an exact decoder by $W : R \to L_{\mathrm{out}}G$. For each basis state,

$$(W \otimes I_B)V|i\rangle = |i\rangle|\eta_i\rangle,$$

because the decoded marginal is pure. Recovery of every matrix unit forces $\langle\eta_j|\eta_i\rangle = 1$, so all environment vectors coincide. The encoding complement is therefore constant.

Conversely, diagonalize

$$\tau_B = \sum_{\beta=1}^{s} \lambda_\beta \mathrm{kb}\,\beta\beta$$

on its support and write

$$V|i\rangle = \sum_{\beta=1}^{s} \sqrt{\lambda}_\beta |u_{i\beta}\rangle_R |\beta\rangle_B.$$

Constancy on matrix units implies

$$\langle u_{j\gamma}|u_{i\beta}\rangle = \delta_{ij}\delta_{\beta\gamma}.$$

Define $T(|i\rangle|\beta\rangle) = |u_{i\beta}\rangle$ and $\sigma = \sum_\beta \lambda_\beta \mathrm{kb}\,\beta\beta$. This gives the normal form and the dimension bound. The displayed decoder is CP, trace preserving, and returns X.

This is the full-code structure underlying exact error correction [3]; it is not a criterion for every smaller observable task.

Approximate correction and replacement benchmarks

Lemma (Recovery from small complementary leakage). *For every complete encoding,*

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \le \min\{a_d, \sqrt{2\delta_{\mathrm{full}}(\mathcal{C})}\}.$$

If $\mathcal{N}$ is replacement, then $\varepsilon_{\mathrm{rec}}(\mathcal{N}) = a_d$. More generally,

$$d_\diamond(\mathcal{N}, \mathcal{T}_{\omega_R}) \le \eta \quad \Rightarrow \quad \varepsilon_{\mathrm{rec}}(\mathcal{N}) \ge \max\{0, a_d - \eta\}.$$

Proof. For any decoded replacement, normalized Bell input produces $I_Q/d \otimes \sigma_L$, whose Bell overlap is d^{-2}. The Bell-projector test therefore gives error at least a_d. Preparing I_L/d attains it: the depolarizing channel is the average of d^2 Weyl conjugations, one of which is identity. This also gives the universal constant-decoder upper bound.

The information–disturbance theorem [4], in full-diamond normalization, gives

$$\frac{1}{4}\Big(\inf_{\mathcal{D}} \|\mathcal{D}\mathcal{N} - \mathrm{id}_L\|_\diamond\Big)^2 \le \inf_{\tau} \|\mathcal{C} - \mathcal{T}_\tau\|_\diamond.$$

Conversion to half-diamond normalization yields . The final assertion follows by decoder contraction and the

reverse triangle inequality.

Proposition (No commuting duplicate logical action). *For $d \geq 2$, two commuting star representations π_1, π_2 cannot both satisfy*

$$\pi_j(X)V = VX \qquad (X \in \mathscr{B}(L)).$$

Proof. For arbitrary X, Y,

$$VXY = \pi_1(X)\pi_2(Y)V = \pi_2(Y)\pi_1(X)V = VYX.$$

Injectivity of V would make the logical matrix algebra commutative.

Classical, hybrid, and discarded interfaces

Theorem (Sharp entirely classical return floor). *Suppose the entire accessible terminal output is classical and fixed auxiliary resources are independent of the unknown input. Every deterministic quantum decoder obeys*

$$d_{\diamond}(\mathscr{D}\mathscr{M}, \mathrm{id}_L) \geq 1 - \frac{1}{d}.$$

The bound is sharp; the full-diamond floor is $2(1 - 1/d)$.

Proof. Complete terminal histories, including failures and stopping symbols, give a POVM:

$$\mathscr{M}(\rho) = \sum_x \mathrm{Tr}\,(E_x\rho)\mathrm{kb}\,xx, \qquad E_x \geq 0, \quad \sum_x E_x = I.$$

A decoder then produces

$$\mathscr{D}\mathscr{M}(\rho) = \sum_x \mathrm{Tr}\,(E_x\rho)\sigma_x.$$

This is measure-and-prepare and hence entanglement breaking [9]. Its Bell-input output is separable. For states a, b,

$$\mathrm{Tr}\,[|\Phi_d\rangle\langle\Phi_d|(a \otimes b)] = \frac{1}{d}\mathrm{Tr}\,(a^{\mathrm{T}}b) \leq \frac{1}{d}.$$

The Bell-projector test proves the lower bound.

For sharpness, measure and reprepare a fixed basis. The dephasing channel is the average of the d phase conjugations, including identity. Separating that term gives an upper bound $(d-1)/d$, matching the lower bound.

Theorem (Exact hybrid teleportation). *For $d = 2^{\ell}$, an independent Bell resource and destructive Bell readout give*

$$\mathscr{M}_{\mathrm{tel}}(X) = \frac{1}{d^2}\sum_{z=1}^{d^2} \mathrm{kb}\, zz_Y \otimes W_z X W_z^{\dagger}.$$

Conditional correction on YE returns the input exactly, including all references. Before the label arrives, the channel to E is replacement by I_E/d. The label uses 2ℓ classical bits. A complete readout dilation has constant complement.

Proof. The coherent Bell transform gives

$$|\psi\rangle \mathrm{longmapsto}\, \frac{1}{d}\sum_z |z\rangle_T W_z |\psi\rangle_E.$$

Dilate destructive readout by $|z\rangle_T \mapsto |z\rangle_Y |z\rangle_G$. Tracing G gives the stated channel. Controlled $W_z^{\dagger}$ returns every matrix unit. Tracing the label instead averages Pauli conjugations and gives I_E/d. Tracing YE gives $\mathrm{Tr}\,(X) I_G/d^2$, so the complement is constant.

This is the standard teleportation mechanism [10]. It is not quantum return through classical records alone: quantum memory E survives.

Proposition (Append-only albums and coherent entropy). *An unedited cumulative classical record satisfies*

$$H(Y_{\le k+1}) = H(Y_{\le k}) + H(Y_{k+1} \mid Y_{\le k}) \ge H(Y_{\le k}).$$

For prescribed-basis readout of P,

$$H(Y) = S(\Delta_P \rho_P) \ge S(\rho_P).$$

Proof. Use the classical chain rule for the first assertion. Dephasing is an average of unitary conjugations, so entropy concavity gives the second.

Proposition (Readout, forgetting, and complete complements). *For a residual factor,*

$$\mathrm{dropBit} \circ \mathrm{Enter}_B = \mathrm{Tr}_B$$

including every surviving receiver and reference. An unreported local CPTP residual operation leaves the accessible channel unchanged.

If a complete residual operation is an isometry $W : B \to B'$, including its measurement environment, then

$$\mathscr{C}' = \mathrm{Ad}_W \mathscr{C}, \qquad \delta_{\mathrm{full}}(\mathscr{C}') = \delta_{\mathrm{full}}(\mathscr{C}).$$

Proof. Both readout-and-forgetting and partial trace sum the diagonal residual blocks. More generally, the adjoint of a trace-preserving local operation maps identity to identity.

Isometric postprocessing preserves referenced trace norms and maps replacement channels to replacement channels. Its CPTP left inverse from Proposition gives the reverse optimized-leakage inequality.

The complete nested finite model

Declare

$$L \cong \mathbb{C}^d, \quad H \cong E \cong \mathbb{C}^h, \quad F_{\mathrm{anc}} \cong \mathbb{C}^f.$$

The symbol F_{anc} denotes a quantum ancilla and is unrelated to the geometric function $F(\eta, r)$.

Before the payload arrives, prepare

$$|\Phi_h\rangle_{HE} = h^{-1/2} \sum_{a=1}^{h} |a\rangle_H |a\rangle_E, \qquad |0\rangle_{F_{\mathrm{anc}}}.$$

All preparations are independent of the payload and its references. The vault dimension is

$$D = dhf.$$

Choose a tensor-cell presentation, sample one Haar unitary $U \in U(D)$, and regard its ideal matrix as known. At cut k, regroup the vault as $P_k B_k$, where

$$r_k = \dim P_k, \qquad b_k = \dim B_k, \qquad r_k b_k = D.$$

The receiver is $R_k = P_k E$. The complete encoding is

$$V_k^U|\psi\rangle = (U \otimes I_E)\big(|\psi\rangle_L |\Phi_h\rangle_{HE} |0\rangle_{F_{\text{anc}}}\big).$$

Set

$$\mathcal{N}_k^U = \mathrm{Tr}_{B_k} \mathrm{Ad}_{V_k^U}, \qquad \mathcal{C}_k^U = \mathrm{Tr}_{R_k} \mathrm{Ad}_{V_k^U}.$$

After this one scrambling operation, release changes control of existing cells only. Already released cells and E undergo no further vault operation. SWAP routing retains its source blanks in complete bookkeeping.

For qubits,

$$d = 2^\ell, \quad h = 2^m, \quad f = 2^a, \quad n = \ell + m + a, \qquad r_k = 2^k, \quad b_k = 2^{n-k}.$$

A cutoff $K < n$ retains a nontrivial residual. The model has the entanglement-assisted retrieval structure compared with Hayden–Preskill retrieval [8]; its unitary is not derived from the spacetime.

The two-copy twirl

Lemma (Haar twirl). *For the full two-copy swap* S,

$$\mathbb{E}_U U^{\dagger \otimes 2} O U^{\otimes 2} = AI + B\mathrm{S},$$

where

$$A = \frac{\mathrm{Tr}\, O - \mathrm{Tr}\,(O\mathrm{S})/D}{D^2 - 1}, \qquad B = \frac{\mathrm{Tr}\,(O\mathrm{S}) - \mathrm{Tr}\, O/D}{D^2 - 1}.$$

For $O = I_{PP'} \otimes \mathrm{S}_{BB'}$, $D = rb$,

$$A = \frac{b(r^2 - 1)}{D^2 - 1}, \qquad B = \frac{r(b^2 - 1)}{D^2 - 1}.$$

Proof. The two-copy average lies in the commutant of $U \otimes U$, which is the span of identity and swap. Trace and swap-trace determine the coefficients. The indicated traces are $r^2 b$ and rb^2.

Complementary Choi moment

Let $|\Phi_d\rangle_{QL} = d^{-1/2}\sum_i |i\rangle_Q|i\rangle_L$. At a cut $D = rb$, define

$$\omega_{QB}^U = (\mathrm{id}_Q \otimes \mathscr{C}^U)(|\Phi_d\rangle\langle\Phi_d|), \qquad \sigma_B^U = \mathscr{C}^U(I_L/d),$$

$$X_{QB}^U = \omega_{QB}^U - I_Q/d \otimes \sigma_B^U, \qquad z(U) = d_\diamond(\mathscr{C}^U, \mathscr{T}_{\sigma_B^U}).$$

Theorem (Exact complementary moment). *For ,*

$$\operatorname{boxed} \mathbb{E}_U \left\| X_{QB}^U \right\|_2^2 = \frac{r(b^2-1)(d^2-1)}{d^2 h(D^2-1)}.$$

Proof. After tracing E, the initial reference–vault state is

$$|\Phi_d\rangle\langle\Phi_d|_{QL} \otimes I_H/h \otimes \mathrm{kb}\,00_{F_{\mathrm{anc}}}.$$

Its purity is $1/h$, its reference purity is $1/d$, and its vault-marginal purity is $1/(dh)$. The swap identity and Lemma give

$$\mathbb{E}\,\mathrm{Tr}\,[(\omega_{QB}^U)^2] = A/d + B/h, \qquad \mathbb{E}\,\mathrm{Tr}\,[(\sigma_B^U)^2] = A + B/(dh).$$

Since $\mathrm{Tr}_Q\,\omega = \sigma$,

$$\|X\|_2^2 = \mathrm{Tr}\,(\omega^2) - d^{-1}\mathrm{Tr}\,(\sigma^2).$$

The A-terms cancel, giving .

Lemma (Choi conversion and leakage moment). *For normalized Choi operators*

$$J_{\mathrm{n}}(\Phi) = (\mathrm{id} \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|),$$

one has

$$\frac{1}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1 \operatorname{led}_\diamond(\Phi,\Psi) \le \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1.$$

Consequently,

$$\mathbb{E}z(U)^2 \le \mathrm{K}(r,b) := \frac{dD(b^2-1)(d^2-1)}{4h(D^2-1)} \le \frac{d^4 f}{4r^2}.$$

Proof. Bell input gives the lower bound. A pure referenced input can be written $(T \otimes I)|\Phi_d\rangle$, where $\mathrm{Tr}\,(T^\dagger T) = d$ and $\|T\|_\infty^2 \le d$. Multiplication on both sides of the Choi difference gives the upper bound; convexity handles mixed inputs.

Since X acts on dimension db,

$$z^2 \le \frac{d^2}{4}\|X\|_1^2 \le \frac{d^3 b}{4}\|X\|_2^2.$$

Insert . Finally use

$$\frac{b^2-1}{D^2-1} \le \frac{b^2}{D^2}, \qquad d^2-1 \le d^2, \qquad D = dhf, \qquad b = D/r.$$

The factor d in the Choi-to-diamond conversion is retained. A Bell-input estimate alone is not a uniform whole-code estimate.

Nested monotonicity

Proposition (Later access cannot worsen the optimum). *Under nested release,*

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{k+1}^U) \le \varepsilon_{\mathrm{rec}}(\mathcal{N}_k^U), \qquad \delta_{\mathrm{full}}(\mathcal{C}_{k+1}^U) \le \delta_{\mathrm{full}}(\mathcal{C}_k^U).$$

The named-center quantity

$$z_k(U) = d_\diamond\Big(\mathcal{C}_k^U, \mathcal{T}_{\mathcal{C}_k^U(I_L/d)}\Big)$$

is also nonincreasing.

At cut zero, for every U,

$$\mathcal{N}_0^U = \mathcal{T}_{I_E/h}, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}_0^U) = \delta_{\mathrm{full}}(\mathcal{C}_0^U) = a_d.$$

Proof. Discarding newly received cells recovers the earlier accessible channel, so an earlier decoder is still available. The later complement is a partial trace of the earlier complement. This sends replacement channels,

including the named center, to their later counterparts.

At cut zero, vault-only evolution leaves the early marginal unchanged. The complement is

$$X\text{longmapsto}\, U(X \otimes I_H/h \otimes \text{kb}\, 00)U^{\dagger},$$

which has a CPTP left inverse obtained by undoing U and tracing the ancillas. Applying that inverse to a replacement comparison gives leakage at least a_d. Comparison with its image of I_L/d gives equality.

Thresholds, a specified decoder, and resource accounting

Theorem (One event for every later retained cut). *Fix* $0 < p < 1$, $\zeta > 0$, *and* $k_* \le K < n$. *If*

$$\text{K}(r_{k_*}, b_{k_*}) \le p\zeta^2,$$

then a Haar family of measure at least $1 - p$ *satisfies*

$$z_k(U) \le \zeta, \qquad \varepsilon_{\text{rec}}(\mathcal{N}_k^U) \le \min\{a_d, \sqrt{2\zeta}\} \quad (k_* \le k \le K).$$

A sufficient condition is

$$r_{k_*} \ge \frac{d^2\sqrt{f}}{2\zeta\sqrt{p}}.$$

For a full-diamond target $0 < \varepsilon < 1$, *take* $\zeta = \varepsilon^2/8$, *obtaining*

$$\text{boxed}\, r_{k_*} \ge \frac{4d^2\sqrt{f}}{\varepsilon^2\sqrt{p}}.$$

Proof. Markov's inequality gives

$$\mathbb{P}\{z_{k_*} > \zeta\} \le \mathbb{E}z_{k_*}^2/\zeta^2 \le p.$$

Nested monotonicity propagates this single device event to every later cut. Lemma gives half-diamond error $\sqrt{2\zeta}$, hence full-diamond error $2\sqrt{2\zeta}$.

This is a device-selection probability, not a decoder success flag, input survival probability, or postselection

probability.

Theorem (Explicit positive-residual qubit window). *Let* $d = 2^{\ell}$, $\ell \geq 1$, *and* $f = 1$. *Fix* $0 < \varepsilon, p < 1$, *integers* $q \geq 1, w \geq 0$, *and set*

$$b_* = 2^{q+w}, \qquad H_* = \frac{4b_* d}{\varepsilon^2 \sqrt{p}}, \qquad h = 2^m, \quad m = \lceil \log_2 H_* \rceil.$$

For $n = \ell + m$, *choose*

$$k_* = n - (q + w), \qquad K = n - q.$$

Then $0 \leq k_* \leq K < n$, *the window has* $w + 1$ *cuts, and a family of measure at least* $1 - p$ *has full-diamond return error at most* ε *throughout the window. The cutoff retains* q *vault qubits.*

Moreover,

$$\frac{4d^2}{\varepsilon^2 \sqrt{p}} \leq r_* < \frac{8d^2}{\varepsilon^2 \sqrt{p}}, \qquad \dim R_* = \frac{dh^2}{b_*} < \frac{64 b_* d^3}{\varepsilon^4 p}.$$

Proof. Rounding gives $H_* \leq h < 2H_*$, so $r_* = dh/b_*$ meets . The cut definitions give the window and residual claims. The remaining bounds follow by substitution.

Example (The sufficient Haar law is not necessary). Take $U = I$, $f = 1$, release L, and retain H, with $h \geq 2$. Then

$$V|\psi\rangle = |\psi\rangle_P |\Phi_h\rangle_{HE}, \qquad R = PE, \quad B = H.$$

Tracing E returns the payload exactly with $r = d$ and a nontrivial residual.

An unambiguous decoder prescription

Use the unnormalized input-first Choi convention

$$J(\Phi) = \sum_{i,j} \mathrm{kb}\, ij \otimes \Phi(\mathrm{kb}\, ij).$$

For a decoder Choi matrix C on $R_*' \otimes L_{\mathrm{out}}$, put

$$\mathscr{D}_C(Y) = \mathrm{Tr}_{R_*'} [(Y^{\mathrm{T}} \otimes I_L) C], \qquad J_{\Delta}(C) = J(\mathscr{D}_C \mathscr{N}_*^U - \mathrm{id}_L).$$

Choose the optimum of

$$\begin{aligned} &ll\text{minimize} && t/2 \\ &\text{subject to} && C \geq 0, \quad \mathrm{Tr}_L C = I_{R_*'}, \\ & && Z \geq J_\Delta(C), \quad Z \geq -J_\Delta(C), \\ & && \mathrm{Tr}_L Z \leq t I_{L'}, \quad t \geq 0, \end{aligned}$$

where C, Z are Hermitian. Among optimal decoder matrices, select the one of least squared Frobenius norm and call its channel $\mathscr{D}_*$.

Theorem (Specified core and fixed later policy). *Program selects a unique CPTP decoder matrix, and its value is* $\varepsilon_{\mathrm{rec}}(\mathcal{N}_*^U)$. *For* $k \geq k_*$, *let* $\Pi_{k \to k_*}$ *discard subsequently received cells while retaining the same E. Then*

$$\mathscr{D}_k = \mathscr{D}_* \Pi_{k \to k_*}$$

satisfies

$$\mathscr{D}_k \mathcal{N}_k^U = \mathscr{D}_* \mathcal{N}_*^U.$$

On the event of Theorem , this channel has half-diamond error at most $\min\{a_d, \sqrt{2\zeta}\}$.

Proof. The first constraints characterize CPTP decoder Choi matrices. For a Hermiticity-preserving map, diamond-norm duality [6] gives

$$\|\Delta\|_\diamond = \min\{t : Z \geq \pm J(\Delta), ?\mathrm{Tr}_{\mathrm{out}} Z \leq t I_{\mathrm{in}}\}.$$

This identifies the objective. The decoder set is compact, and the diamond error is continuous and convex. Its minimizer set is compact and convex; the strictly convex secondary objective selects a unique decoder matrix.

Nested release gives $\Pi_{k \to k_*} \mathcal{N}_k^U = \mathcal{N}_*^U$, proving .

The prescription depends on the known device, not on the unknown input or a favorable record. It is not an efficiency theorem. Retained cuts are alternative stopping interfaces, not repeated returns of copies of one unknown payload.

Conservative receiver workspace

Let $A_* = \dim R_*$. If the decoder Choi rank is g, then

$$g \le dA_*, \qquad W_{\mathscr{D}} : R_* \to L_{\text{out}} G, \quad \dim G = g$$

is a Stinespring isometry. For qubit dimensions, put

$$q_k = \log_2 \dim R_k, \quad q_* = \log_2 A_*, \quad q_G = \lceil \log_2 g \rceil.$$

Allocating fresh output and environment gives the conservative count

$$Q_{\text{receiver, peak}} \le q_k + \ell + q_G \le q_k + q_* + 2\ell.$$

This follows by diagonalizing the decoder Choi matrix and extending the resulting isometry to a unitary on a padded space.

The count excludes supplier residuals, external references, clocks, routing buffers, and retired stores. Finite continuous one-qubit gates and CNOT suffice for exact finite qubit synthesis [7]; no efficient gate count is asserted.

Coherent entropy turnover with accurate recovery

Fix a pure payload independently of U. After tracing E,

$$\rho_{\text{vault}}^{U} = h^{-1} U P_\psi U^\dagger, \qquad \operatorname{rank} P_\psi = h.$$

This is a mixed random-projector ensemble. The entropy below is that of P_k, not automatically that of $P_k E$. All logarithms are base two.

Theorem (Quantum and record purity). *At $D = rb$, define*

$$p_{\text{q}} = \frac{r(b^2-1) + b(r^2-1)/h}{D^2-1}, \qquad p_{\text{c}} = \frac{Db + (D-b)/h - 1}{D^2-1}.$$

Then

$$\mathbb{E}\,\mathrm{Tr}\,[(\rho_P^U)^2] = p_{\mathrm{q}}, \qquad \mathbb{E}\sum_y (p_y^U)^2 = p_{\mathrm{c}},$$

where p_y^U *are prescribed-basis measurement probabilities. For* $t = \min(r, bh)$,

$$-\log_2 p_{\mathrm{q}} \le \mathbb{E}S(P) \le \log_2 t,$$

$$-\log_2 p_{\mathrm{c}} \le \mathbb{E}H(Y) \le \log_2 r,$$

and

$$0 \le \log_2 t - \mathbb{E}S(P) \le \log_2\left(1 + \frac{\min(r, bh)}{\max(r, bh)}\right) \le 1.$$

Proof. Twirl $\mathrm{S}_{PP'} \otimes I_{BB'}$. Its identity and full-swap coefficients are

$$\frac{r(b^2-1)}{D^2-1}, \qquad \frac{b(r^2-1)}{D^2-1}.$$

The input purity is $1/h$, giving p_{q}.

The averaged two-copy vault state is

$$\frac{1 - 1/(hD)}{D^2-1} I + \frac{1/h - 1/D}{D^2-1} \mathrm{S}.$$

For the rank-b projector $\Pi_y = \mathrm{kb}\, yy_P \otimes I_B$, this gives

$$\mathbb{E}(p_y^U)^2 = \frac{b^2[1 - 1/(hD)] + b[1/h - 1/D]}{D^2-1}.$$

Sum over r outcomes.

The pure complete state on PBE bounds the rank of ρ_P by $\min(r, bh)$. Entropy is at least minus log purity, and Jensen's inequality gives the lower mean bounds. Finally, $p_{\mathrm{q}} \le 1/r + 1/(bh)$, which gives the stated gap.

The balance is r versus bh, not generally r versus b. Using a pure Haar-state entropy formula when $h > 1$ would describe the wrong ensemble.

Theorem (Scaled strict turnover and one-qubit cutoff). *Let*

$$d = 4^{s}, \qquad s \geq 2, \qquad f = 1, \qquad h = 2^{m}, \qquad h \geq \frac{2d^{2}}{\zeta}.$$

Choose

$$k_{*} = m + s - 1, \quad k_{\mathrm{peak}} = m + s, \quad k_{+} = m + s + 1, \quad K = m + 2s - 1.$$

A Haar family of measure at least $1/(4d)$ *satisfies:*

1. $z_{k}(U) \leq \zeta$ *for every* $k_{*} \leq k \leq K$*;*

2. *the fixed decoder policy has half-diamond error at most* $\min\{a_{d}, \sqrt{2\zeta}\}$*;*

3. *for the fixed pure payload,*

$$S(P^{U}_{k_{\mathrm{peak}}}) > \log_{2}(h\sqrt{d}/2) \geq \max\{S(P^{U}_{k_{*}}), S(P^{U}_{k_{+}}), S(P^{U}_{K})\};$$

4. B_{K} *is one physical qubit. The ensemble mean also has a strict peak relative to* k_{*} *and* k_{+}*.*

Proof. Put $t = \sqrt{d} = 2^{s}$. The three relevant cut dimensions are

$$(r_{*}, b_{*}) = (ht/2, 2t), \quad (r_{\mathrm{peak}}, b_{\mathrm{peak}}) = (ht, t), \quad (r_{+}, b_{+}) = (2ht, t/2),$$

and $b_{K} = 2$. Equation gives

$$\mathbb{E}z_{*}^{2} \leq d^{3}/h^{2}, \qquad \mathbb{P}\{z_{*} > \zeta\} \leq \frac{1}{4d}.$$

At the peak, set $v = ht/2$. The purity formula gives

$$vp_{\mathrm{q}}(ht, t, h) = \frac{dh^{2}(2d - 1) - d}{2(d^{2}h^{2} - 1)},$$

and hence

$$1 - vp_{\mathrm{q}} = \frac{dh^{2} + d - 2}{2(d^{2}h^{2} - 1)} > \frac{1}{2d}.$$

Markov's inequality implies that

$$\mathbb{P}\{\mathrm{Tr}[(\rho^{U}_{P_{\mathrm{peak}}})^{2}] < 1/v\} > \frac{1}{2d}.$$

On that event, $S(P^{U}_{\mathrm{peak}}) > \log_2 v$. The rank envelopes at k_* and k_+ are v; at K the envelope is $2h \le v$. Subtracting the leakage-failure set leaves measure greater than $1/(4d)$. Monotonicity and the decoder theorem finish the same-device claims.

Finally $p_{\mathrm{q}} < 1/v$ puts the peak mean above $\log_2 v$, whereas the neighboring means are at most that value.

Corollary (Full-diamond turnover). *For* $0 < \varepsilon < 1$, *take*

$$\zeta = \varepsilon^2/8, \qquad h = 2^m \ge \frac{16d^2}{\varepsilon^2}.$$

The family in Theorem has full-diamond return error at most ε at every retained cut.

Proof. Use $2\sqrt{2\zeta} = \varepsilon$.

Corollary (Dimension sixteen). *For* $d = 16, f = 1, h = 2^m$, *the retained cuts*

$$k_* = m + 1, \qquad k_{\mathrm{peak}} = m + 2, \qquad K = m + 3$$

have

$$(r_*, b_*) = (2h, 8), \quad (r_{\mathrm{peak}}, b_{\mathrm{peak}}) = (4h, 4), \quad (r_K, b_K) = (8h, 2).$$

If $h \ge 512/\zeta$, *a family of measure at least* $1/64$ *has* $z_k \le \zeta$, *one decoder core, and*

$$S(P^{U}_{k_{\mathrm{peak}}}) > \log_2(2h) \ge \max\{S(P^{U}_{k_*}), S(P^{U}_{K})\}.$$

For full-diamond target ε, it suffices that $h \ge 4096/\varepsilon^2$.

Example (Retained parameter ledger). Take

$$\varepsilon = 1/5, \quad \zeta = 1/200, \quad m = 17, \quad h = 131072.$$

The required h is at least 102400. The vault has 21 qubits and the early register has 17.

$c\|c\|c\|c$cut	late qubits	residual qubits	receiver qubits
hline 18	18	3	35
19	19	2	36
20	20	1	37

The full-diamond allowance is 0.2. The decoder input dimension is 2^{35}, its Choi rank is at most 2^{39}, and gives a conservative final-cut receiver peak of 80 qubits. Clocks, routing buffers, retired stores, and the supplier's residual are separate entries.

These are parameter calculations, not an executed Haar sample or decoder optimization.

Hosting the protocol during the dynamical phase

Assumption (Ideal controls on the evolving host). Within a regular laboratory region of $(\widetilde{M}, g)$, permit:

1. finitely many finite-dimensional registers, including several registers at one laboratory;

2. prescribed local unitaries, independent blank preparations, and complete destructive instruments;

3. coherent identity storage along declared timelike laboratory worldlines;

4. identity quantum wires and classical record wires along declared future-causal routes;

5. sufficient time, storage, buffers, and fresh resources for the finite schedule. Auxiliary inputs are independent of the unknown payload unless included in it explicitly. Every inaccessible output is accounted for. Finite registers are treated as test devices: their gravitational backreaction is not included in the source .

These are implementation assumptions, not consequences of Einstein's equation. No gate-time optimum, energy bound, bandwidth theorem, or continuum field realization is asserted.

Lemma (Routing and fixed spectators). *SWAP transport into independent blanks implements identity transport including all references. Vacated sources are fixed blanks.*

Appending fixed input-independent factors and applying output isometries does not change optimal recovery error, optimized complementary leakage, or named-center leakage, provided the original interface is recoverable by a CPTP reduction.

Proof. For one factor,

$$\mathrm{SWAP}\,(|\psi\rangle_A|0\rangle_{A'}) = |0\rangle_A|\psi\rangle_{A'}.$$

Linearity extends this identity to every matrix unit and reference.

For spectators and isometries, one direction follows by CPTP postprocessing. Discarding spectators and using a CPTP isometric left inverse gives the other. The named center undergoes the same fixed transformation.

Theorem (Dynamic causal-hosting theorem). *Fix a finite qubit instance of , a known ideal U, a finite retained window, and the decoder core of Theorem . Under Assumption , the evolving spacetime admits a finite scheduled realization with:*

1. *independent resource preparation before payload injection;*

2. *future-causal routes for every nonlocal dependency;*

3. *accessible channel exactly $\mathcal{N}_k^U$ at every designated retained cut, after fixed port identifications;*

4. *encoding complement $\mathcal{C}_k^U$ up to isometries and input-independent fixed factors;*

5. *the same fixed later decoder policy and decoded channel ;*

6. *independent destructive clocks with the proper-time calibration of Section . The realization need not be confined to a settled static patch. In particular, it can include a nonzero payload-storage interval in the exterior transition region where is nonzero.*

Proof. Choose a Euclidean convex laboratory region away from the core and two fixed-coordinate laboratories inside it. Prepare the Bell resource and blanks sufficiently early at a common laboratory, then distribute *H* and *E* along causal routes before payload injection. The early spacetime is Minkowskian, and there is no preparation deadline.

Choose a supplier radius $r_s > r_1$ sufficiently large that

$$r_s/c + \tau_f/2 > \tau_a.$$

At the supplier, there is an open time interval with $0 < \eta - r_s/c < \tau_f$, during which the curvature witness is nonzero. Inject the payload there and store it coherently for a nonzero subinterval. This identity storage is an explicit part of

the assumed implementation, not an inference from local flatness.

Apply a finite local realization of U. Then release the fixed cells in order and route them to the receiver. For each route, choose a sufficiently late arrival so that holds. The number of routes is finite, so all required times are finite.

Store E and all received cells. No further vault operation acts on released cells. At the snapshot following the k-th arrival, the accessible quantum channel is exactly $\mathcal{N}_k^U$. Source placeholders are fixed blanks and are included in the complete bookkeeping. Lemma preserves the encoding diagnostics.

At any selected stopping cut, discard arrivals later than k_* and apply the fixed decoder core. This realizes . Decoder environments are distinguished from the predecoder residual.

Finally append independent clocks, calibrated to coordinate or proper time as declared. Corollary leaves the payload channel unchanged. Their complete environments are fixed independent additions.

Remark (Why no automatic "geometric noise" appears). An exact identity wire is an assumption about a channel along a causal route. A time-dependent metric does not contradict that assumption, and therefore does not automatically add an error term to the ideal finite construction.

For physical carriers, curvature, expansion, acceleration, redshift, and imperfect control may change the implemented channel. Those effects must be calculated from a specified propagation and detector model. They are addressed conditionally in Section .

Scheduled ancestry and initially entangled inputs

Theorem (Causal ancestry bounds influence). *For the complete hosted circuit, pullback of any terminal observable is supported only on its backward scheduled ancestors. Their events lie in the metric causal past of the terminal event.*

An unreported trace-preserving intervention outside that ancestry cannot change the terminal reduced output, even for initially entangled inputs.

Proof. Work backward through the circuit. A channel disjoint from the current observable support acts on

identity, and its adjoint is unital. A channel meeting the support adds only its input factors. Induction proves the support assertion.

Every dependency edge is future causal, so transitivity gives the metric assertion. Outside the pulled-back support, a trace-preserving intervention leaves identity unchanged. No product-state assumption is used.

Postselection and reported outcomes are different interfaces. The theorem concerns unreported trace-preserving interventions.

The finite certificate still does not determine geometry

The same finite gates and channels can be scheduled for different choices of $\beta, \lambda, \tau_a, \tau_f$. The uniform sufficient routing rule remains available. Waiting times can also be enlarged without changing a channel matrix.

Consequently the return certificate alone does not determine the deficit, expansion history, physical travel duration, or classical source. Unlike the original static host, this manuscript supplies one explicit source pair, but that pair is not reconstructed from the recovery certificate.

Transferred tasks, privacy, and composition

Theorem (Tasks inherited from a return certificate). *If*

$$d_{\diamond}(\Lambda, \mathrm{id}_L) \le e,$$

then:

1. *every referenced input changes by trace distance at most e;*

2. *a Bell input has returned Bell overlap at least* $1 - e$*;*

3. *every orthonormal basis message can be read after return with error at most e;*

4. *every CPTP task* $\mathscr{A}$ *satisfies* $d_{\diamond}(\mathscr{A}\Lambda, \mathscr{A}) \le e$*;*

5. *an effect probability changes by at most e, and a Hermitian observable expectation changes by at most* $2e\|O\|_{\infty}$.

Proof. Apply the definition of stabilized diamond distance, its contraction under CPTP postprocessing, and trace/operator-norm duality. Bell and correct-message projectors are effects.

Theorem (Privacy of the declared encoding residual). *If*

$$d_{\diamond}(\mathscr{C}, \mathscr{T}_{\tau_B}) \leq \zeta,$$

then every referenced input obeys

$$d_{\mathrm{tr}}((\mathrm{id}_Q \otimes \mathscr{C})(\rho_{QL}), \rho_Q \otimes \tau_B) \leq \zeta.$$

For unreferenced inputs,

$$d_{\mathrm{tr}}(\mathscr{C}(\rho), \mathscr{C}(\sigma)) \leq \min\{1, 2\zeta\}.$$

With equal priors, a residual-only observer distinguishes them with success probability at most $\min\{1, 1/2 + \zeta\}$. *Residual postprocessing cannot improve these bounds.*

Proof. Use the diamond comparison, the triangle inequality through τ_B, the equal-prior discrimination formula, and CPTP contraction.

Additional transcripts, receiver registers, or decoder environments require an enlarged adversary model. They do not automatically inherit the same leakage allowance.

Independent sessions and relays

Theorem (Independent-resource convoy). *Suppose the actual joint returned channel is* $\mathrm{bigotimes}_{i=1}^{N} \Lambda_i$, *with* $d_{\diamond}(\Lambda_i, \mathrm{id}_{L_i}) \leq e_i$. *Then*

$$d_{\diamond}\left(\mathrm{bigotimes}_i \Lambda_i, \mathrm{id}_{\mathrm{bigotimes}_i L_i}\right) \leq \min\{1, \sum_i e_i\}.$$

Payloads may be mutually entangled and entangled with a reference. If device events fail with probabilities at most p_i, *their conjunction has probability at least* $1 - \sum_i p_i$, *when positive.*

Proof. Replace tensor factors by identity one at a time. Channels have diamond norm one, so the triangle

inequality gives the bound, already including references. The probability claim is the union bound.

Independent resource preparation and noninteraction are sufficient for the tensor-product hypothesis. Good marginal tests alone are not.

For N fresh uses of one selected device and total full-diamond target ε, certify per-session target ε/N. For $f = 1$, a sufficient dimension is

$$r_* \geq \frac{4d^2N^2}{\varepsilon^2\sqrt{p}}.$$

One successful device-selection event works on all fresh resource copies; resources are not freely recycled.

For serial decoded links,

$$d_\diamond(\Lambda_J \cdots \Lambda_1, \mathrm{id}_L) \leq \min\left\{1, \sum_j d_\diamond(\Lambda_j, \mathrm{id}_L)\right\}.$$

The same telescoping proof applies. Each hosted link needs its own causal routes, storage, and timing.

Complete reusable-memory reset

Theorem (Uniform session-and-reset contract). *Let M contain every memory capable of affecting a later session, and fix a reset state ω_M. Suppose*

$$\mathscr{F}_i : \mathscr{B}(L_i M) \to \mathscr{B}(L_i' M)$$

includes the session, decoding, and reset, and on the entire input space satisfies

$$d_\diamond\left(\mathscr{F}_i, \mathrm{id}_{L_i} \otimes \mathscr{T}_{\omega_M}\right) \leq u_i.$$

Previously returned payloads are isolated from later operations. Then the sequential channel differs from identity return on all payloads followed by replacement of M by ω_M by at most

$$\min\{1, \sum_i u_i\}$$

in half-diamond distance, including arbitrary initial memory correlations.

Proof. Insert identity wires for prior outputs and future inputs. Replace each actual session by its ideal session-and-reset map, one at a time. Pre- and post-composition contract diamond distance. The ideal sequence returns the original joint payload marginal and replaces the memory, removing its initial correlations.

A correct reset marginal on one tested memory input is weaker than this contract. Small previous logical leakage does not establish resource freshness or reusable entanglement. Discarded systems must not reinteract unless included in M.

Implementation errors on the dynamic geometry

Theorem (Access-preserving implementation headroom). *After common access-preserving output identifications, suppose*

$$d_{\diamond}(\mathscr{V}_{\mathrm{act}}, \mathscr{V}_{\mathrm{ideal}}) \le \eta_{\mathrm{enc}}, \qquad d_{\diamond}(\mathscr{D}_{\mathrm{imp}}, \mathscr{D}_{\mathrm{ideal}}) \le \iota.$$

If the ideal half-diamond return error is at most e, then

$$d_{\diamond}(\mathscr{D}_{\mathrm{imp}}\mathscr{N}_{\mathrm{act}}, \mathrm{id}_L) \le \min\{1, e + \eta_{\mathrm{enc}} + \iota\}.$$

If the complete encoding comparison includes the actual full residual and the ideal complement is within ζ of replacement, the actual optimized leakage is at most $\min\{1, \zeta + \eta_{\mathrm{enc}}\}$. *For actual and ideal named centers,*

$$z_{\mathrm{act}} \le \min\{1, z_{\mathrm{ideal}} + 2\eta_{\mathrm{enc}}\}.$$

Proof. Partial trace contracts the complete encoding comparison. Insert the ideal channel and decoder in the triangle inequality.

For optimized leakage, compare with the ideal replacement state. For the named center, evaluating the channel difference on I_L/d adds at most η_{enc} to the center-state comparison, in addition to the channel comparison itself.

An accessible-channel comparison alone does not control every actual residual output.

For full target ε, reserve

$$\eta_{\mathrm{enc}} + \iota < \varepsilon/2, \qquad u_{\mathrm{err}} = \varepsilon/2 - \eta_{\mathrm{enc}} - \iota > 0.$$

Choosing $\zeta = u_{\mathrm{err}}^{2}/2$ gives the sufficient threshold

$$r_{*} \geq \frac{d^{2}\sqrt{f}}{u_{\mathrm{err}}^{2}\sqrt{p}}.$$

Preparation, storage, routing, local controls, readout, and clock control must be charged wherever they are approximated.

A physical map is needed before assigning geometry an error

A metric difference is not a diamond distance. To turn geometry or calibration uncertainty into a channel estimate, one must specify the carrier Hilbert space, Hamiltonian, coupling, route, detector, and access-preserving output identification.

For example, suppose two descriptions on a common finite Hilbert space have Hamiltonian generators $K(\eta)$ and $K_{0}(\eta)$, both expressed in the same conformal-time parameter. Duhamel's identity gives

$$\|U - U_{0}\|_{\infty} \leq \frac{1}{\hbar}\int \|K(\eta) - K_{0}(\eta)\|_{\infty}\, \mathrm{d}\eta.$$

Consequently,

$$d_{\diamond}(\mathrm{Ad}_{U}, \mathrm{Ad}_{U_{0}}) \leq \min\left\{1, \frac{1}{\hbar}\int \|K - K_{0}\|_{\infty}\, \mathrm{d}\eta\right\}.$$

This becomes a geometric error estimate only after a model specifies how K depends on the geometry and controls.

Example (Proper-time phase calibration). A fixed-coordinate qubit with proper-time Hamiltonian

$$H_{\mathrm{proper}} = \frac{\hbar\omega}{2}Z$$

has conformal-time generator $K(\eta) = a(\eta)H_{\mathrm{proper}}$. If an uncompensated calibration uses a_{est}, then

$$\delta\tau = \int (a - a_{\mathrm{est}})\, \mathrm{d}\eta$$

produces a phase-channel difference

$$d_{\diamond}(\mathrm{Ad}_{U}, \mathrm{Ad}_{U_{\mathrm{est}}}) = \left|\sin\left(\frac{\omega\delta\tau}{2}\right)\right| \le \min\{1, |\omega\delta\tau|/2\}.$$

A known phase can instead be corrected by the allowed controls. Thus evolution of proper time need not itself cause irreversible logical error.

For a complete finite network, compatible gate, instrument, and wire replacements with half-diamond errors γ_v give total error at most $\sum_v \gamma_v$ by telescoping. A complete-residual claim requires complete comparisons, not only visible output tests.

Finite descriptions and strict margins

A generic exact Haar unitary has ideal real parameters and is not a finite-bit object. The decoder SDP is likewise a mathematical selection rule, not a claimed hardware computation.

Proposition (Finite-description approximations). *If a selected device has strict return headroom and a strict finite-cut entropy gap, sufficiently accurate finite-description approximations of its encoding and decoder preserve the target and the strict gap.*

Proof. For unitaries on a common space,

$$d_{\diamond}(\mathrm{Ad}_{U}, \mathrm{Ad}_{U'}) \le \|U - U'\|_{\infty}.$$

Expand the difference of conjugations into two terms to obtain this bound, including references.

A finite continuous-gate decomposition exists [7]. Approximating its finitely many angles by rational multiples of π gives finite descriptions arbitrarily close in operator norm. Do the same for a unitary extension of the decoder isometry. Theorem preserves the return target within headroom.

The finitely many reduced-state matrices and their eigenvalues vary continuously. Since $-x\log_2 x$, extended by zero at $x = 0$, is continuous, strict finite-cut entropy inequalities persist under sufficiently small perturbations.

This is an existence statement, not a report of a successful search, compiler run, fault-tolerant implementation, or

numerical experiment.

Separation from the I-D Host-A pulse shell

New in this revision. This dynamical cone is the closest explicit dynamical cone in the Hawk07 family. It is *not* Host A and *not* Host B of I-D [16]. Both constructions exhibit an outgoing NEC violation; they are *not the same shell*.

Inverse Einstein stays a source ledger, not a matter theory. No Host C is created. Indexed citations: ID-v5 [16], IC-v2 [17], IBG [18].

Standing ledger. Host A and Host B stay separate. Host C is not created. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Dom (*T*) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. No Einstein dynamics, derived metric, spacetime emergence, Page curve, island, bounce, vacuum-energy selection, or de Sitter state is claimed as an Enter theorem. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. wired=false.

Consolidated result and remaining physical obligations

Theorem (Dynamic-spacetime recovery certificate). *The construction in this manuscript has the following properties.*

1. *The metric is a smooth, genuinely nonstationary Lorentzian metric on a spacetime with its regular axis included. The full spacetime is globally hyperbolic.*

2. *Its string-induced deformation has retarded radial support. Every bounded exterior region becomes exactly isometric to a static conical product after its explicit settling time.*

3. *Equations – provide its exact Einstein source. This source is conserved, has a positive string-like late core, and violates the null energy condition in its outgoing transition. No independent microscopic matter model is claimed.*

4. *For every qubit payload dimension d, target $0 < \varepsilon < 1$, and $0 < p < 1$, there is a finite positive-residual retained window with a Haar family of measure at least $1 - p$ and full-diamond return error at most ε. The sufficient released dimension is*

$$r_* \geq \frac{4d^2\sqrt{f}}{\varepsilon^2\sqrt{p}}.$$

5. *One specified decoder core, followed by the fixed discard policy, gives the identical decoded channel at every later retained cut. Receiver dimension and workspace are charged separately from released dimension.*

6. *For $d = 16$, a family of measure at least $1/64$ combines accurate return, strict same-device three-cut coherent entropy turnover for a fixed pure input, and a one-qubit final residual. For $d = 4^s$, $s \geq 2$, the scaled measure is at least $1/(4d)$.*

7. *Under the stated ideal controls, the finite interface has a causal realization on the evolving metric, including payload presence during the nonstationary phase. Independent destructive clock readout does not alter the logical channel or consume time.*

8. *Entirely classical return has sharp half-diamond floor $1 - 1/d$. Hybrid teleportation is different because quantum memory survives. Append-only classical record entropy does not turn over.*

9. *Return tasks, declared-residual privacy, independent-resource convoys, complete reset sessions, and relays inherit the stated diamond allowances.*

10. *Physical geometric implementation errors require an additional propagation and detector model. Once an access-preserving channel comparison is supplied, the error allowances add as in . No part of this theorem constructs a geometric throat, traversable wormhole, gravitational island, material spaceship transport, evaporation endpoint, or quantum gravitational backreaction.*

Proof. The metric, source, and causal claims follow from Propositions –, Theorem , and Proposition . The finite claims follow from the moment, conversion, threshold, decoder, and turnover results. The dynamic hosting theorem supplies the conditional realization. The classical, privacy, composition, and approximation results give

the remaining assertions.

The excluded gravitational and microscopic identifications are absent from the construction and are not supplied by the finite certificate.

What a conventional matter-model successor must supply

To replace the effective source by an ordinary cosmic-string formation theory, a successor must provide:

1. an independently specified matter action or constitutive model;

2. consistent initial data and a matter evolution law;

3. an Einstein–matter solution, or a controlled perturbative or numerical approximation;

4. energy conditions, stability, and the relevant fluid or field propagation cones;

5. a justified treatment of the core and any thin-string limit;

6. physical register, carrier, detector, and control models;

7. access-preserving approximation estimates linking those models to the finite channel construction. Local conservation of G/κ is not a substitute for these requirements.

Separate energy and reset ledgers

A reset channel does not specify its heat and work bill. SWAP with an independent blank resets a qubit for any chosen common excitation gap $\Delta > 0$. For input excitation probability p_{ex}, the unreturned carrier receives energy Δp_{ex}, although the observed reset channel is unchanged when Δ is changed. Restoring the consumed blank is a further task.

The classical geometric source, finite logical leakage, device energy, and thermal reset implementation are therefore separate data.

Conclusion. The conversion produces an explicitly time-dependent spacetime, rather than inserting time labels into the old static cone. Its metric, curvature, Einstein source, causal front, global hyperbolicity, and late conical

region are all stated and checked. The exact formation source is deliberately identified as engineered and energy-condition violating.

The finite recovery certificate remains a quantum-information contract. Its conditional realization can overlap genuine spacetime evolution, but its existence does not identify quantum recovery with a geometric wormhole or turn an inverse Einstein source into a microscopic cosmic-string theory.

appendix

Analytic and documentary separations

The original manuscript discusses free Green operators, selected continuum states, and a characteristic-product programme. Those objects do not enter the finite encoding or the dynamic Einstein calculation here.

A retarded Green operator, a Hadamard state, a finite decoder, and a joint distributional contraction are different mathematical types. The smooth globally hyperbolic host improves the causal setting for a separately specified normally hyperbolic field theory, but it does not itself specify a state, detector, interacting coefficient, or field-to-register encoding.

In particular, this paper does not construct the joint characteristic map, its section and remainder, transverse estimates, contact compatibilities, output wavefront theorem, or normalized coefficient admission discussed documentarily in the supplied source. No claimed analytic admission exit is opened by changing the metric.

Proposition (Reconstruction descent). *Let $\mathcal{P}$ be a vector space of presentations,* $\mathrm{Rec} : \mathcal{P} \to \mathcal{K}$ *a surjective linear reconstruction, and $A : \mathcal{P} \to \mathcal{V}$ linear. There is a unique linear $\overline{A} : \mathcal{K} \to \mathcal{V}$ satisfying $A = \overline{A}\mathrm{Rec}$ if and only if*

$$\ker \mathrm{Rec} \subseteq \ker A.$$

Agreement after restriction to a smaller open domain establishes only the corresponding restricted condition.

Proof. A factorization annihilates $\ker \mathrm{Rec}$. Conversely, two presentations with the same reconstruction differ by an element of that kernel, making $\overline{A}(\mathrm{Rec}P) = A(P)$ well defined. Surjectivity gives uniqueness. Restriction cannot

detect distributions supported outside its domain.

Lemma (A representative-dependent cancellation). *In one normal variable,*

$$q\delta'(q) + \delta(q) = 0.$$

Thus, if a reconstruction convention sends M_R to $\delta(q) \otimes R$, sends $i\theta M_R$ to $\delta'(q) \otimes R$, and respects multiplication by q, then

$$\mathrm{Rec}((qi\theta + 1)M_R) = 0.$$

For $R \neq 0$, the two summands are separately nonzero.

Proof. For a test function φ,

$$\langle q\delta', \varphi\rangle = -\frac{\mathrm{d}}{\mathrm{d}q}(q\varphi(q))\Big|_{q=0} = -\varphi(0).$$

Tensor with R; separated tests establish the final assertion.

These elementary facts do not construct the missing joint analytic object, prove a universal obstruction, or admit an interacting quantum coefficient.

Debugging and submission ledger

begingroup 1.15 Corrections enforced in the dynamic revision.

toprule Potential error Correction

midrule endfirsthead toprule Potential error Correction

midrule endhead bottomrule endfoot

Calling a static cone with time labels a dynamic spacetime. The metric contains retarded formation and a changing scale factor. Its curvature pulse is explicit, and an everywhere timelike Killing field is excluded.

Claiming that $\alpha(T)r$ requires matter only near the axis. Its Einstein tensor contains off-axis flux and pressure. The

retarded finite-core metric replaces that ansatz.

Presenting $T = G/\kappa$ as a solved microscopic matter theory. It is an exact inverse Einstein source construction. No independent matter action or initial-value theorem is claimed.

Hiding the energy-condition cost of the exact formation model. An explicit null vector gives negative $T_{\mu\nu}k^{\mu}k^{\nu}$. The outgoing transition compensates the positive core.

Calling a smooth transition a distributional shock. The switches are smooth. No shock or vacuum-wave interpretation is assumed.

Importing a sound horizon without a fluid model. No sound speed or perfect-fluid equation of state is specified. Only the metric's radial null front is used.

Treating arbitrary-tension strings as pure cones. The ordinary formation comparison is perturbative, and $q \neq 1$ also produces a gravitational potential.

Assuming FLRW expansion becomes static merely near a string. Exact settling requires the explicit constant late scale factor used here. Persistent expansion is a different case.

Claiming global settling at one finite time. Only bounded exterior regions settle at their stated finite times. The front continues outward at all finite times.

Equating a smooth included core with an excised apex. The full regular-core spacetime is globally hyperbolic. Deleting its axis produces a different, non-globally-hyperbolic spacetime.

Confusing conformal time and laboratory proper time. For a fixed-coordinate laboratory, $d\tau = a(\eta)d\eta$. Clock calibration states which time is being labelled.

Treating destructive clock readout as deletion of time. Only the finite clock interface is consumed. The metric and its coordinates remain.

Automatically adding a "geometric error" to an ideal identity wire. An error requires a physical implementation map. Ideal causal identity wires preserve the abstract channel exactly.

Inferring a diamond bound directly from a metric norm. A carrier, Hamiltonian, coupling, route, detector, and output identification must first be specified.

Applying the classical floor to a hybrid receiver. The entire accessible terminal output must be classical.

Teleportation retains quantum memory.

Dropping the dimension factor in Choi conversion. The factor d is retained in the whole-code diamond estimate.

Calling the sufficient Haar threshold necessary. An identity encoding returns exactly with released dimension d.

Counting only released dimension. Receiver dimension is $r_* h$; workspace, buffers, clocks, and supplier residuals have separate ledgers.

Treating later cuts as repeated copies of one return. They are alternative stopping interfaces.

Transferring the measure 1/64 unchanged to all dimensions. The scaled turnover allowance is $1/(4d)$.

Using pure-state Haar entropy with an entangled early register. The vault is a rank-h projector divided by h. The mixed-projector moment is used.

Claiming reset from a correct memory marginal. A uniform complete payload–memory contract is required.

Ignoring changes in the named leakage center. Optimized leakage costs at most one encoding allowance. The actual named center can cost a second.

Calling exact Haar parameters finite-bit hardware data. Ideal parameters are separated from finite-description approximations with strict error headroom.

Assigning finite registers an undeclared gravitational source. The registers are test devices. No finite-density-matrix-to-stress-tensor map is supplied.

Treating source citations as independent proof verification. The supplied manuscript is documentary. External tools are identified, and the new calculations are explicit.

endgroup

Single-file compilation. This source uses an internal bibliography, standard packages, and no external figures or data files. Save it as Hawk07-C-dynamic.tex and run pdfLaTeX twice to resolve the table of contents, citations, and cross-references. No successful compilation or automated symbolic verification is asserted.

Declarations

Mathematical scope. The new geometric content is an explicit smooth time-dependent host, its computed Einstein source, causal support, global causal analysis, and the revised dynamic hosting theorem. The finite recovery and turnover ingredients are retained and rederived from the supplied manuscript. No global priority claim is made.

Physical scope. The exact transition source violates energy conditions and has no specified microscopic realization. The model is not claimed to be an observationally established cosmic-string formation process. No quantum stress tensor, register backreaction, or geometric wormhole is constructed.

Implementation scope. Controls, storage, and identity wires are ideal assumptions. The work reports no laboratory experiment, executed simulation, decoder optimization, or physical compiler run.

Authorship. The supplied author field is retained as editorial metadata. Its retention does not establish contribution, consent, or submission authorization. Final authorship and submission responsibility require confirmation by the named author.

Use of artificial intelligence. An AI assistant assisted with restructuring, mathematical consistency checking, source consultation, and preparation. No proof-assistant certification, referee verification, or journal acceptance is asserted.

References

[1] N. Cheung, Hawk07-C: Finite Enter analogues on a prescribed conical host (parent static sheet), Cheung Ngo Medical Limited (2026). Source record; not a premise.

[2] M. Lake and T. Suyama, "Evolution of FLRW spacetime after the birth of a cosmic string." *Physical Review D* **85** (2012), 083521. doi:10.1103/PhysRevD.85.083521. https://arxiv.org/abs/1112.2478.

[3] E. Knill and R. Laflamme, "Theory of quantum error-correcting codes." *Physical Review A* **55** (1997), 900–911. doi:10.1103/PhysRevA.55.900. https://arxiv.org/abs/quant-ph/9604034.

[4] D. Kretschmann, D. Schlingemann, and R. F. Werner, "The information-disturbance tradeoff and the continuity of Stinespring's representation." *IEEE Transactions on Information Theory* **54** (2008), 1708–1717. doi:10.1109/TIT.2008.917696. https://arxiv.org/abs/quant-ph/0605009.

[5] J. Watrous, *The Theory of Quantum Information*. Cambridge University Press, 2018. doi:10.1017/9781316848142. Author's manuscript: https://jhwatrous.github.io/TQI.pdf.

[6] J. Watrous, "Simpler semidefinite programs for completely bounded norms." *Chicago Journal of Theoretical Computer Science* **2013**, Article 8. https://arxiv.org/abs/1207.5726.

[7] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, "Elementary gates for quantum computation." *Physical Review A* **52** (1995), 3457–3467. doi:10.1103/PhysRevA.52.3457. https://arxiv.org/abs/quant-ph/9503016.

[8] P. Hayden and J. Preskill, "Black holes as mirrors: Quantum information in random subsystems." *Journal of High Energy Physics* **2007**(09), 120. doi:10.1088/1126-6708/2007/09/120. https://arxiv.org/abs/0708.4025.

[9] M. Horodecki, P. W. Shor, and M. B. Ruskai, "Entanglement breaking channels." *Reviews in Mathematical Physics* **15** (2003), 629–641. doi:10.1142/S0129055X03001709. https://arxiv.org/abs/quant-ph/0302031.

[10] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels." *Physical Review Letters* **70** (1993), 1895–1899. doi:10.1103/PhysRevLett.70.1895.

[11] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026), https://doi.org/10.6084/m9.figshare.34335024.v1.

[12] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026), https://doi.org/10.6084/m9.figshare.34549575.v1.

[13] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026), https://doi.org/10.6084/m9.figshare.34335582.v1.

[14] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026), https://doi.org/10.6084/m9.figshare.34441758.v1.

[15] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026), https://doi.org/10.6084/m9.figshare.34442907.v1.

[16] N. Cheung, ID-v5 Dynamical Spacetime Enter over Two Prescribed Hosts, in *The Philosophy of Enter*, Cheung Ngo Medical Limited (2026), p. 128. Source record; not a premise.

[17] N. Cheung, IC-v2: The Shredder Interface, in *The Philosophy of Enter*, Cheung Ngo Medical Limited (2026), p. 99. Source record; not a premise.

[18] N. Cheung, IBG: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime, in *Green Operators and Finite Instruments on a Prescribed Conical Spacetime*, Cheung Ngo Medical Limited (2026). Source record; not a premise.

[19] N. Cheung, IB-ECHO: Diffracted echoes and finite supplied-report instruments, in *Green Operators and Finite Instruments on a Prescribed Conical Spacetime*, Cheung Ngo Medical Limited (2026). Source record; not a premise.